Use the chain rule to find the derivative of each of the following functions. Remember your exponent and logarithmic laws, these will help you.

Chain Rule	$\frac{d}{dt}(f(g(t))) = g'(t) \cdot f'(g(t))$

In words, we differentiate the "big function" with respect to the independent variable, then plug in the "small function" and then multiply the result by the derivative of the "small function"

Example 1:

Find the derivative of $y = (t+1)^4$.

Observe that if $f(t) = t^4$ and $g(t) = t+1$, then $y = f(g(t))$. So then,

$$f(t) = t^4 \Longrightarrow f'(t) = 4t^3 \Longrightarrow f'(g(t)) = 4g(t)^3 = 4(t+1)^3$$
$$g(t) = t+1 \Longrightarrow g'(t) = 1$$

So then $y' = g'(t)f'(g(t)) = 1 \cdot 4(t+1)^3 = 4(t+1)^3$.

Example 2:

Find the derivative of $y = e^{-6x}$.

Observe that if $f(x) = e^x$ and $g(x) = -6x$, then $y = f(g(x))$. So then,

$$f(x) = e^x \Longrightarrow f'(x) = e^x \Longrightarrow f'(g(x)) = e^{g(t)} = e^{-6x}$$
$$g(x) = -6x \Longrightarrow g'(x) = -6$$

So then $y' = g'(x)f'(g(x)) = (-6) \cdot e^{-6x} = -6e^{-6x}$.

Example 3:

Find the derivative of $w = 5(2z+3)^2$.

Observe that if $f(z) = 5z^2$ and $g(z) = 2z+3$, then $w = f(g(z))$. So then,

$$f(z) = 5z^2 \Longrightarrow f'(z) = 10z \Longrightarrow f'(g(z)) = 10g(z) = 10(2z+3)$$
$$g(z) = 2z+3 \Longrightarrow g'(z) = 2$$

So then $w' = g'(z)f'(g(z)) = 2 \cdot 10(2z+3) = 20(2z+3)$.

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1. $f(x) = (x+1)^{99}$
2. $g(x) = (4x^2+1)^7$
3. $w = (t^2+1)^{100}$
4. $R = (q^2+1)^4$
5. $w = (5r-6)^3$
6. $f(x) = (x^3+x^2)^{-90}$
7. $y = 12 - 3x^2 + 2e^{3x}$
8. $y = \sqrt{s^3+1}$
9. $f(x) = 6e^{5x} + e^{-x^2}$
10. $C = 12(3q^2-5)^3$
11. $w = e^{-3t^2}$
12. $y = 5e^{5t+1}$
13. $y = \ln(5t+1)$
14. $w = e^{\sqrt{s}}$
15. $f(t) = \ln(t^2+1)$
16. $f(x) = \ln(1-x)$
17. $f(x) = \ln(e^x+1)$
18. $f(x) = \ln(1-e^{-x})$
19. $f(x) = \ln(\ln(x))$
20. $f(x) = (\ln(x))^3$
21. $y = 5 + \ln(3t+2)$
22. $y = (5+e^x)^2$
23. $y = 5x + \ln(x+2)$
24. $y = \sqrt{e^x+1}$
25. $P = (1+\ln(x))^{0.5}$
26. $f(t) = (e^t+e^{-t})^{-1}$
27. $f(x) = \sqrt{2+\sqrt{x}}$

Answers

1. $f'(x) = 99(x+1)^{98}$
2. $g'(x) = 56x(4x^2+1)^6$
3. $w' = 200t(t^2+1)^{99}$
4. $R' = 8q(q^2+1)^3$
5. $w' = 15(5r-6)^2$
6. $f'(x) = -90(3x^2+2x)(x^3+x^2)^{-91}$
7. $y' = -6x + 6e^{3x}$
8. $y' = \frac{3}{2}s^2(s^3+1)^{-1/2}$
9. $f'(x) = 30e^{5x} - 2xe^{-x^2}$
10. $C' = 216q(3q^2-5)^2$
11. $w' = -6te^{-3t^2}$
12. $y' = 25e^{5t+1}$
13. $y' = \frac{5}{5t+1}$
14. $w' = \frac{1}{2}s^{-1/2}e^{\sqrt{s}}$
15. $f'(t) = \frac{2t}{t^2+1}$
16. $f'(x) = -\frac{1}{1-x}$
17. $f'(x) = \frac{e^x}{e^x+1}$
18. $f'(x) = \frac{xe^{-x}}{1-e^{-x}}$
19. $f'(x) = \frac{1}{x\ln(x)}$
20. $f'(x) = \frac{3}{x}(\ln(x))2$
21. $y' = \frac{3}{3t+2}$
22. $y' = 2e^x(5+e^x)$
23. $y' = 5 + \frac{1}{x+2}$
24. $y' = \frac{1}{2}e^x(e^x+1)^{-1/2}$
25. $P' = \frac{1}{2x}(1+\ln(x))^{-1/2}$
26. $f'(t) = -(e^t - e^{-t})(e^t + e^{-t})^{-2}$
27. $f'(x) = \frac{1}{4}x^{-1/2}(2+\sqrt{x})^{-1/2}$

The End.